Review of Regularization Techniques in Electrocardiographic Imaging

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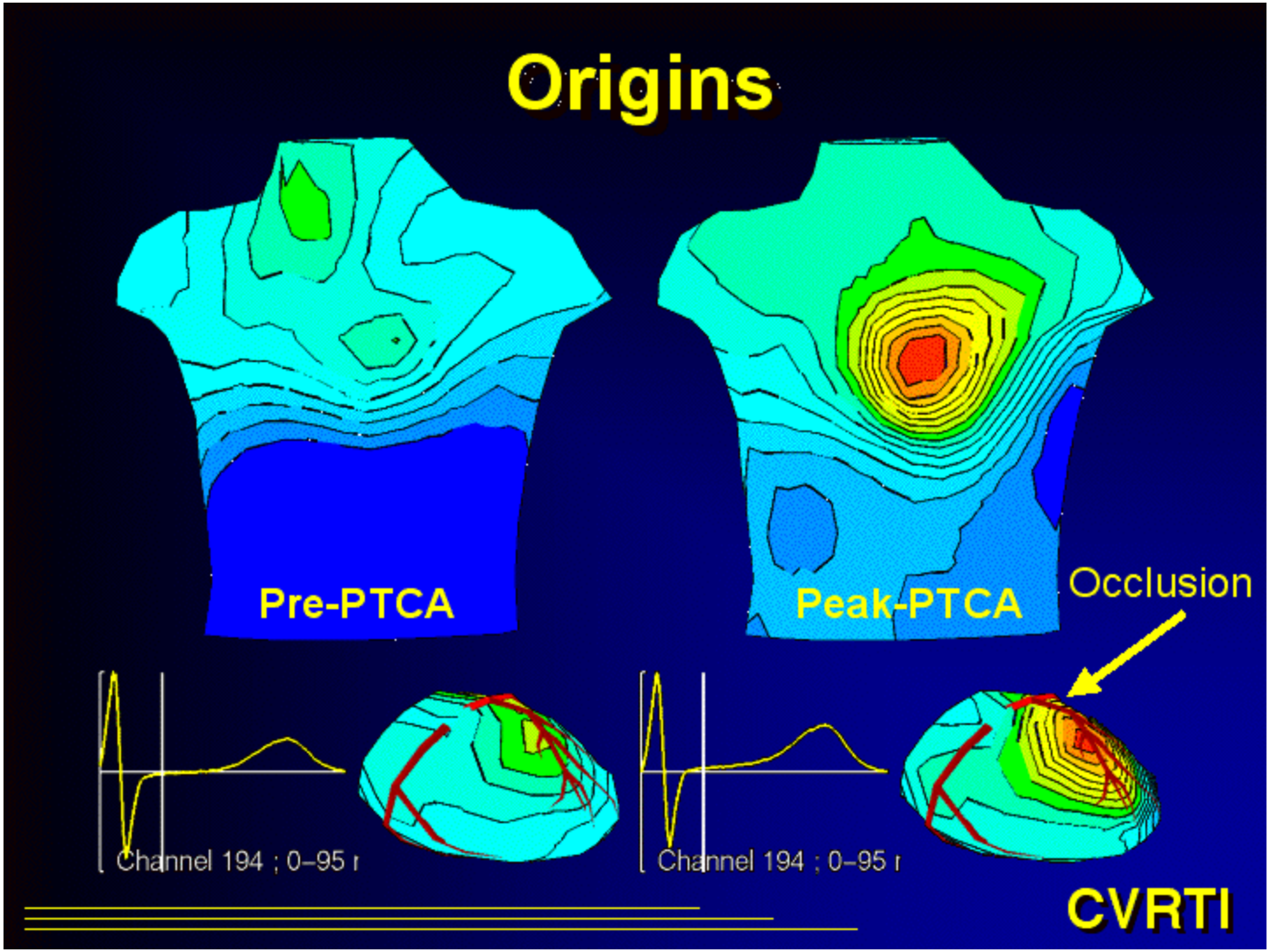
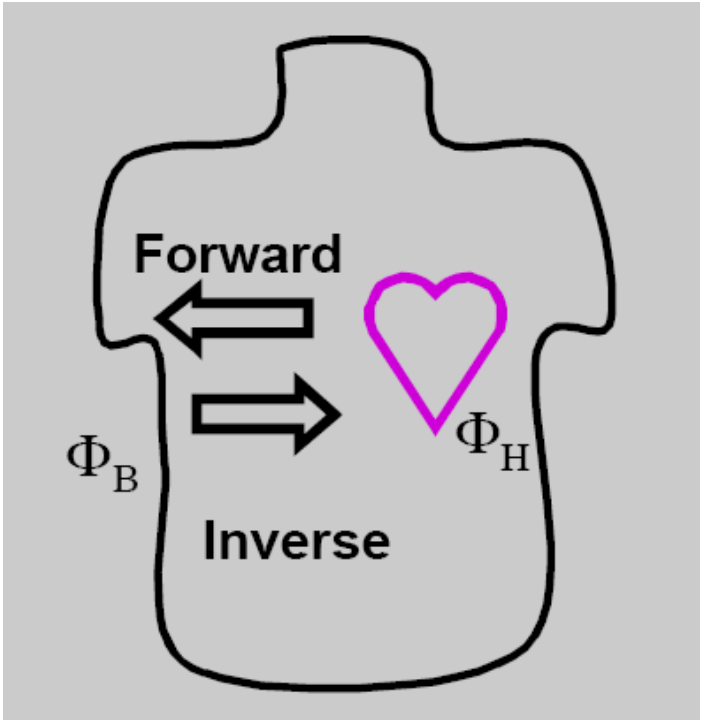
1. ECGI Problem

ECGI = “heart as the potential distribution on the epicardium”



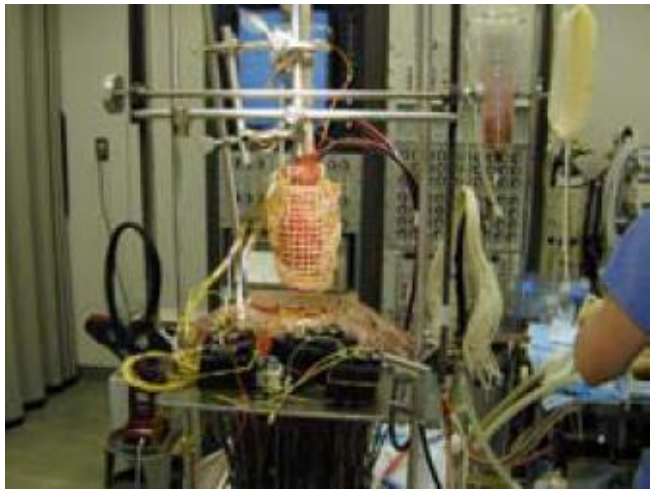
ECGI speaks mathematical language: $\Phi_B = A\,\Phi_H$

- Technical prerequisites:
  - Problem formulation in terms of differential equations
  - Numerical solution techniques
    - Boundary element method – BEM
    - Finite element method – FEM
  - **Regularization of the inverse solution**
    - **ECGI is an ill-posed problem**

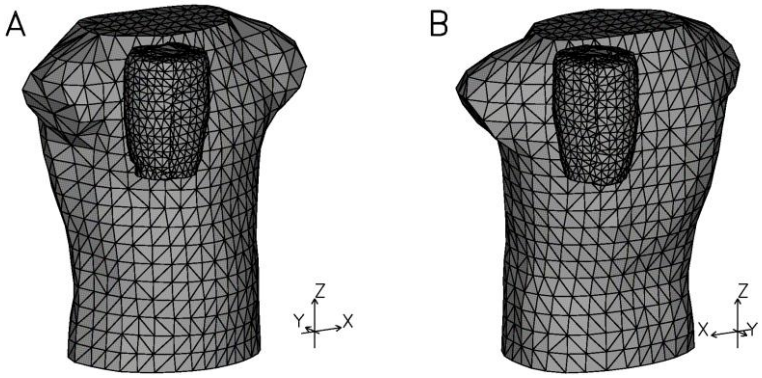


Motivation: Comparing various regularization techniques using the same volume conductor and cardiac source models

- Step 1:
  - Measuring “cage” potentials at 602 leads
  - Perfused canine heart; sinus rhythm
  - 1000 Hz sampling rate; 4-7 sec recordings



- Step 2:
  - Computing “body surface” potentials at 771 nodes
  - BEM



Regularization techniques in a nutshell

- Tikhonov-based regularizations (Group A)

$$\min_{\Phi_H} \{ \| \Phi_B - A\Phi_H \|_2 + \lambda^2 \| \Lambda\Phi_H \|_2 \}$$

λ – regularization parameter

Λ – regularization operator (Z=I, F=G, S=L)

- Iterative methods (Group B)

- Non-quadratic methods (Group C)

$$\min_{\Phi_H} \{ \| \Phi_B - A\Phi_H \|_2 + \lambda^2 \| \Lambda\Phi_H \|_1 \}$$

13 regularization techniques

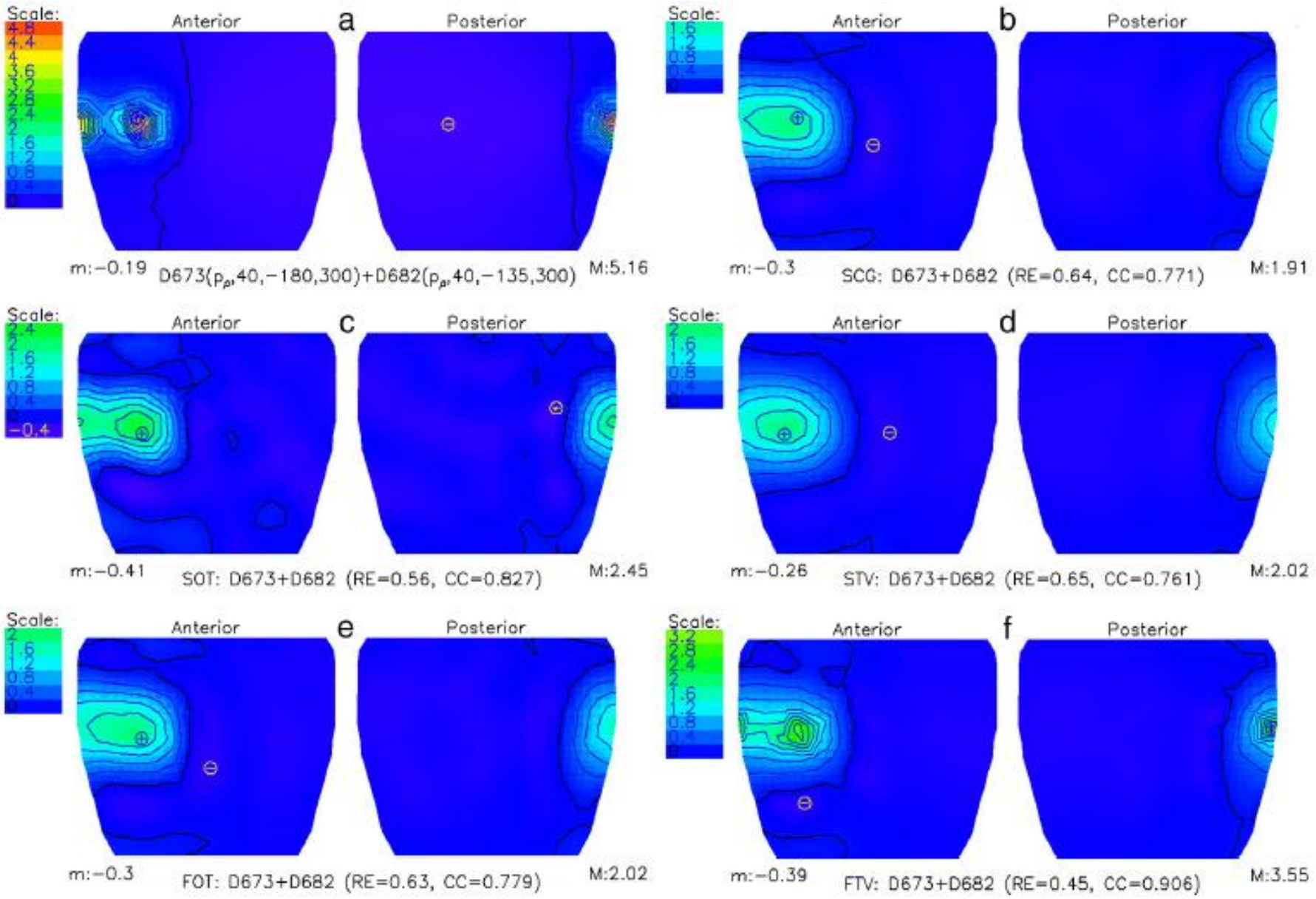
Group	Acronym	Short description
	ZOT	Zero-order Tikhonov
A	FOT	First-order
	SOT	Second-order
	ZTSVD	Zero-order truncated singular value decomposition
	FTSVD	First-order
	STSVD	Second-order
B	ZCG	Zero-order conjugate gradient
	FCG	First-order
	SCG	Second-order
	v-method	
	MINRES	
C	FTV	Total variation
	STV	Total variation with Laplacian

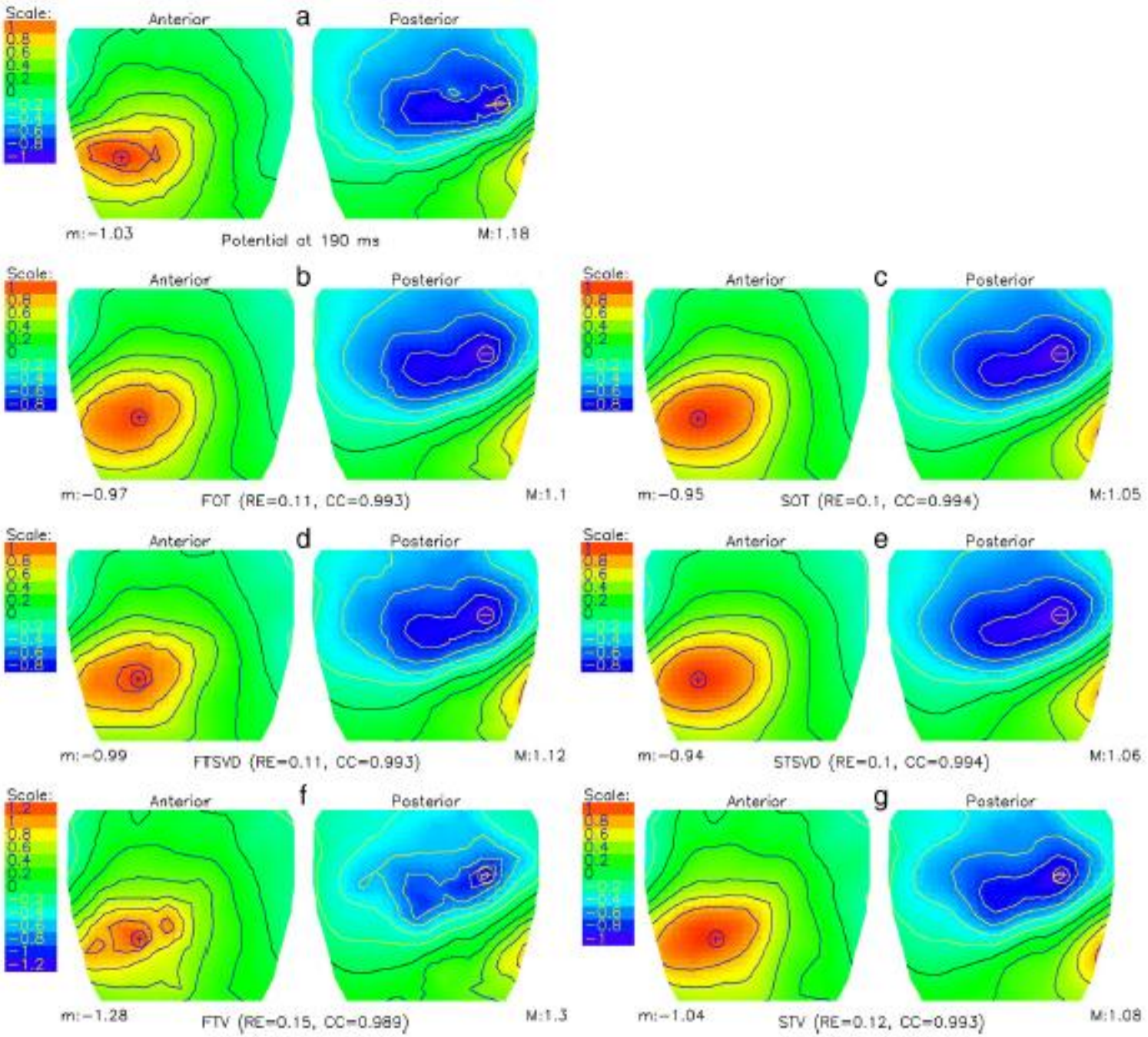
2. Evaluation

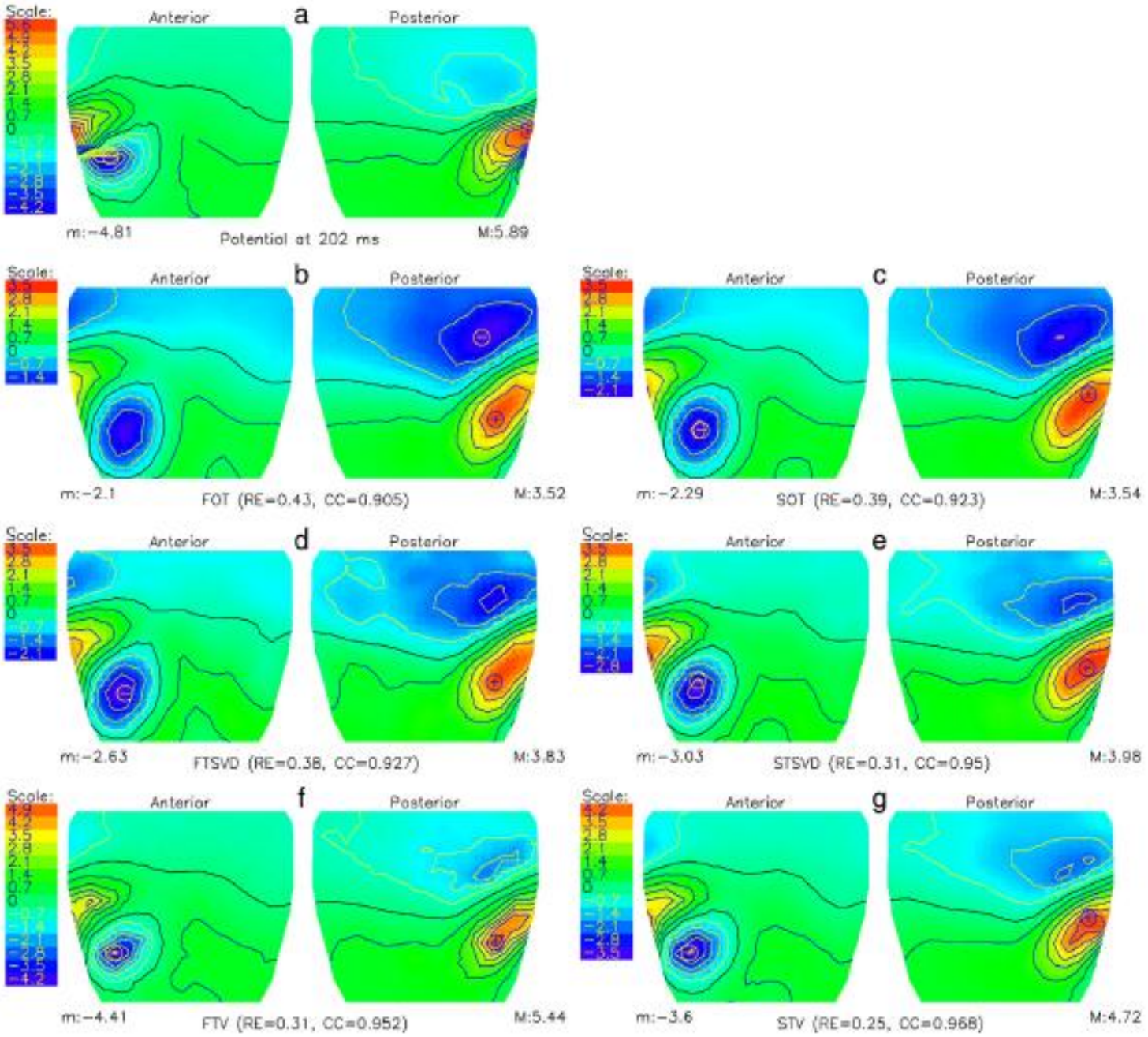
Key Questions

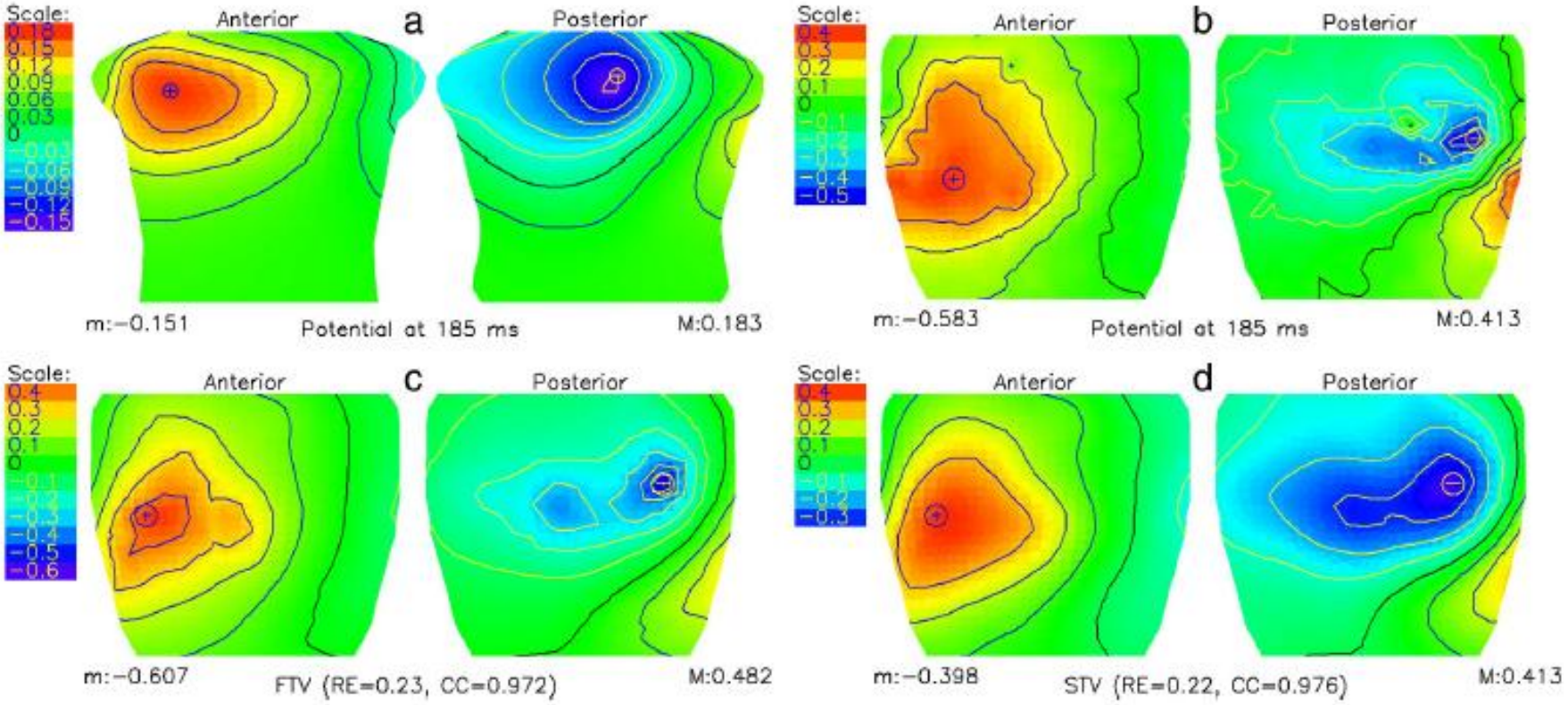
KQ #1: Group A vs. Group B vs. Group C

KQ #2: Z vs. F vs. S

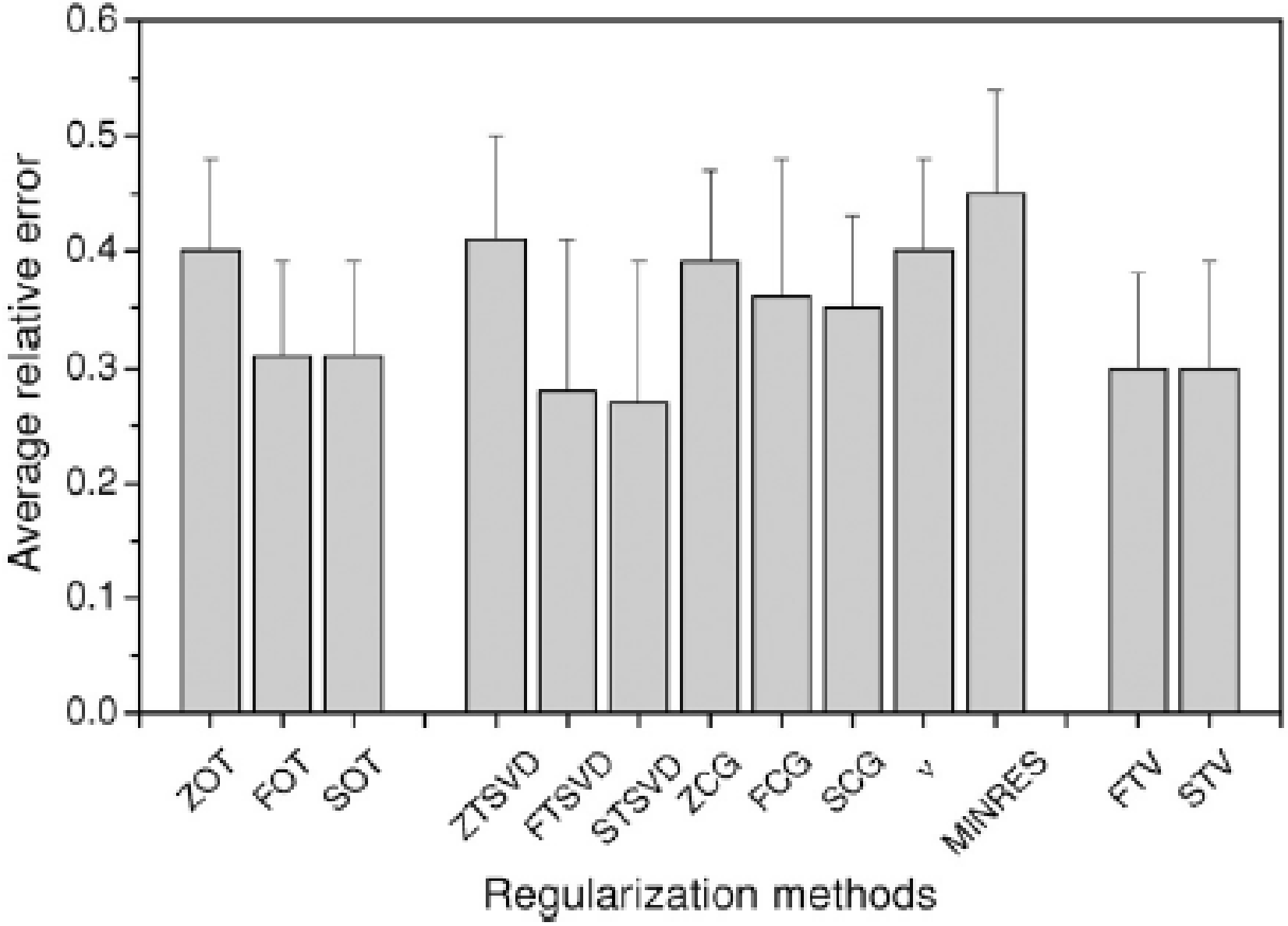

Scale:
Anterior
a
Posterior
m:−0.19
D673(p_ρ,40,−180,300)+D682(p_ρ,40,−135,300)
M:5.16
Scale:
Anterior
b
Posterior
m:−0.3
SCG: D673+D682 (RE=0.64, CC=0.771)
M:1.91
Scale:
Anterior
c
Posterior
m:−0.41
SOT: D673+D682 (RE=0.56, CC=0.827)
M:2.45
Scale:
Anterior
d
Posterior
m:−0.26
STV: D673+D682 (RE=0.65, CC=0.761)
M:2.02
Scale:
Anterior
e
Posterior
m:−0.3
FOT: D673+D682 (RE=0.63, CC=0.779)
M:2.02
Scale:
Anterior
f
Posterior
m:−0.39
FTV: D673+D682 (RE=0.45, CC=0.906)
M:3.55


Scale:
Anterior
a
Posterior
m:−1.03
Potential at 190 ms
M:1.18
b
m:−0.97
FOT (RE=0.11, CC=0.993)
M:1.1
c
m:−0.95
SOT (RE=0.1, CC=0.994)
M:1.05
d
m:−0.99
FTSVD (RE=0.11, CC=0.993)
M:1.12
e
m:−0.94
STSVD (RE=0.1, CC=0.994)
M:1.06
f
m:−1.28
FTV (RE=0.15, CC=0.989)
M:1.3
g
m:−1.04
STV (RE=0.12, CC=0.993)
M:1.08


Scale:
Anterior
a
Posterior
m:−4.81
Potential at 202 ms
M:5.89
Scale:
Anterior
b
Posterior
m:−2.1
FOT (RE=0.43, CC=0.905)
M:3.52
Scale:
Anterior
c
Posterior
m:−2.29
SOT (RE=0.39, CC=0.923)
M:3.54
Scale:
Anterior
d
Posterior
m:−2.63
FTSVD (RE=0.38, CC=0.927)
M:3.83
Scale:
Anterior
e
Posterior
m:−3.03
STSVD (RE=0.31, CC=0.95)
M:3.98
Scale:
Anterior
f
Posterior
m:−4.41
FTV (RE=0.31, CC=0.952)
M:5.44
Scale:
Anterior
g
Posterior
m:−3.6
STV (RE=0.25, CC=0.968)
M:4.72


Scale:
Anterior
a
Posterior
m:−0.151
Potential at 185 ms
M:0.183
Scale:
Anterior
b
Posterior
m:−0.583
Potential at 185 ms
M:0.413
Scale:
Anterior
c
Posterior
m:−0.607
FTV (RE=0.23, CC=0.972)
M:0.482
Scale:
Anterior
d
Posterior
m:−0.398
STV (RE=0.22, CC=0.976)
M:0.413

KQ #2


Average relative error
0.0
0.1
0.2
0.3
0.4
0.5
0.6
ZOT
FOT
SOT
ZTSVD
FTSVD
STSVD
ZCG
FCG
SCG
MINRES
FTV
STV
Regularization methods

3. Conclusions

Key take-aways

- Little difference among three main groups of regularization techniques
- FTV tends to under-regularize the inverse solution

Strengths	Limitations	Future work
▪ Sound physiological model of the heart ▪ Unified simulation framework ▪ Comprehensive evaluation of regularization methodologies	▪ Cage potentials were recorded at a distance from the epicardium and have therefore somewhat smoothed-out patterns ▪ Body surface potentials were computed (rather than measured) ▪ Sinus rhythm	▪ Epicardial sock data ▪ Pacing, sites of early activation ▪ Infarcted hearts